Name ______________________ Class ____________ Date ________

6-5

Video Tutor

Multiplying Polynomials

Going Deeper

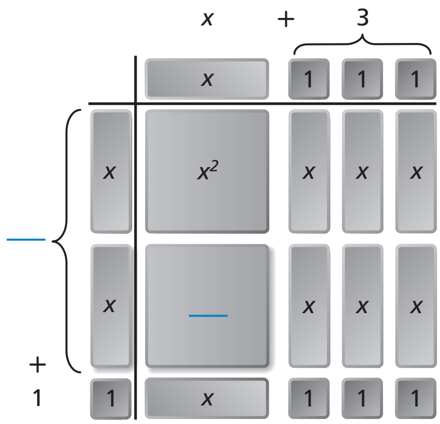
Essential question: *How do you multiply polynomials?*

A **monomial** is a number, a variable, or the product of a number and one or more variables raised to whole number powers, such as 5, x, $-8y$, and $3x^2y^4$. A **polynomial** is a monomial or a sum of monomials. Each monomial in the expression is called a **term**. A polynomial with two terms is a **binomial**. You can multiply two binomials by using algebra tiles.

CC.9–12.A.SSE.2

1 EXPLORE Multiplying Two Binomials Using Algebra Tiles

To use algebra tiles to multiply $(2x + 1)(x + 3)$, first represent $2x + 1$ vertically along the left side of an algebra tile diagram and $x + 3$ horizontally along the top. Then use x^2-tiles, x-tiles, and 1-tiles to complete the diagram, as shown below.



$2x(x + 3) =$ ____ $x^2 +$ ____ x

$1(x + 3) =$ ____ $x +$ ____

____ $x^2 +$ ____ $x +$ ____

$(2x + 1)(x + 3) =$ ____ $x^2 +$ ____ $x +$ ____

The product is a **trinomial**, a polynomial with three terms.

REFLECT

1a. Look at the algebra tile diagram. What two terms in the original binomials combine to form the x^2-term in the trinomial? How do they combine (by multiplying, by adding, or by subtracting)?

____

1b. Look at the algebra tile diagram. What two terms in the original binomials combine to form the constant term in the trinomial? How do they combine (by multiplying, by adding, or by subtracting)?

____

1c. Look at the algebra tile diagram. Show how the terms of the original binomials combine to form the *x*-term in the trinomial.

1d. You can verify that the expressions are equivalent by substituting a value for *x* into both expressions and simplifying to show that they are equal. Verify that the expressions are equivalent. Use $x = 4$.

1e. Suppose you want to use algebra tiles to find the product $(2x + 1)(x + 2)$. Describe how you can modify the algebra tile diagram to find the product.

1f. Suppose you want to use algebra tiles to find the product $(2x + 2)(x + 3)$. Describe how you can modify the algebra tile diagram to find the answer.

CC.9–12.A.APR.1

2 ENGAGE Multiplying Binomials Using the Distributive Property

Using algebra tiles to multiply two binomials is a useful tool for understanding how the two binomials are being multiplied. However, it is not a very practical method for everyday use. Using the distributive property is.

To multiply $(2x + 1)(x + 3)$ using the distributive property, you distribute the binomial $x + 3$ to each term of $2x + 1$. Then you distribute the monomial $2x$ to each term of $x + 3$ as well as the monomial 1 to each term of $x + 3$.

$$\begin{aligned}(\mathbf{2x + 1})(x + 3) &= \mathbf{2x}(x + 3) + \mathbf{1}(x + 3)\\ &= 2x^2 + 6x + x + 3\\ &= 2x^2 + 7x + 3\end{aligned}$$

Notice that the product found using algebra tiles in the Explore is the same as the product found here using the distributive property. Thus, the two methods are equivalent.

To multiply $(4x - 7)(3x + 6)$ using the distributive property, you should think of $4x - 7$ as $4x + (-7)$ and therefore keep the negative sign with the 7.

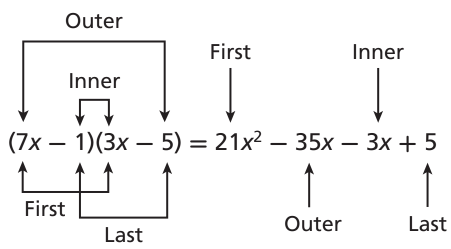
$$\begin{aligned}(\mathbf{4x - 7})(3x + 6) &= \mathbf{4x}(3x + 6) - \mathbf{7}(3x + 6)\\ &= 12x^2 + 24x - 21x - 42\\ &= 12x^2 + 3x - 42\end{aligned}$$

This method of using the distributive property to multiply two binomials is referred to as the FOIL method. The letters of the word FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast and will help you remember how to use the distributive property to multiply binomials.

You apply the FOIL method by multiplying each of the four pairs of terms described below and then simplifying the resulting polynomial.

- **F**irst refers to the first terms of each binomial.
- **O**uter refers to the two terms on the outside of the expression.
- **I**nner refers to the two terms on the inside of the expression.
- **L**ast refers to the last terms of each binomial.

Now multiply $(7x - 1)(3x - 5)$ using FOIL. Again, think of $7x - 1$ as $7x + (-1)$ and $3x - 5$ as $3x + (-5)$. This results in a positive constant term of 5 because $(-1)(-5) = 5$.



$$(7x - 1)(3x - 5) = 21x^2 - 38x + 5$$

Notice that the trinomials are written with variable terms in descending order of exponents and with the constant term last. This is a standard form for writing polynomials: Starting with the variable term with the greatest exponent, write the other variable terms in descending order of their exponents, and put the constant term last.

REFLECT

2a. Refer back to the Explore. Using the tiles, you multiplied $2x$ by $(x + \square)$ and then multiplied 1 by $(x + \square)$. You are using the ________________ property.

2b. In FOIL, which of the products combine to form the x-term?

____

2c. In FOIL, which of the products combine to form the constant term?

____

2d. In FOIL, which of the products combine to form the x^2-term?

____

2e. Two binomials are multiplied to form a trinomial. When is the constant term of the trinomial positive? When is it negative?

____

____

CC.9–12.A.APR.1

3 EXAMPLE Multiplying Two Binomials Using FOIL

Multiply $(12x - 5)(3x + 6)$ using the FOIL method.

First Inner

$$(12x - 5)(3x + 6) = \square x^2 + \square x - \square x - \square$$

Outer Last

$(12x - 5)(3x + 6) =$ ______________________

REFLECT

3a. How does the final x-term in the answer to the Example relate to your answer to Question 2b? Explain.

____

____

3b. How does the final constant in the answer to the Example relate to your answer to Question 2c? Explain.

____

3c. How does the final x^2-term in the answer to the Example relate to your answer to Question 2d? Explain.

____

3d. Suppose the problem in the example were $(12x - 5)(3x + 2)$. Would the x^2-term in the product change? Would the x-term change? Would the constant term change? Explain your reasoning.

____

____

____

____

____

____

3e. Multiply $(12x - 5)(3x + 2)$.

____

As with binomials, you can multiply two polynomials by using the distributive property so that every term in the first factor is multiplied by every term in the second factor. You also use the product of powers property ($a^m \cdot a^n = a^{m+n}$) each time you multiply two terms.

CC.9–12.A.APR.1

4 EXAMPLE Multiplying Polynomials

Find the product.

A $(4x^2)(2x^3 - x^2 + 5)$

$= (4x^2)(2x^3) + (4x^2)(-x^2) + (4x^2)(5)$ Distributive property

$= 8x^5 -$ ________ $+$ ________ Multiply monomials.

B $(x - 3)(-x^2 + 2x + 1)$

Method 1: Use a horizontal arrangement.

$(x - 3)(-x^2 + 2x + 1)$

$= x(-x^2) + x(2x) + x(1) - 3(-x^2) - 3(2x) - 3(1)$ Distribute x and then -3.

$= -x^3 +$ ______ $+ x +$ ______ $-$ ______ $- 3$ Multiply monomials.

$= -x^3 +$ ______ $-$ ______ $-$ ______ Combine like terms.

Method 2: Use a vertical arrangement.

$-x^2 \quad + \quad 2x \quad + \quad 1$ Write the polynomials vertically.

$x \quad - \quad 3$

$3x^2 \quad - \quad 6x \quad - \quad 3$ Multiply $(-x^2 + 2x + 1)$ by -3.

$-x^3 +$ ______ $+$ ______ Multiply $(-x^2 + 2x + 1)$ by x.

$-x^3 +$ ______ $-$ ______ $-$ ______ Add.

REFLECT

4a. Is the product of two polynomials always another polynomial? Explain.

____

____

4b. If one polynomial has m terms and the other has n terms, how many terms does the product of the polynomials have before it is simplified?

____

PRACTICE

Find each product.

1. $(x + 2)(x + 3)$

2. $(x + 7)(x + 11)$

3. $(2x + 13)(x - 6)$

4. $(2x - 5)(3x + 1)$

5. $(2x^3)(2x^2 - 9x + 3)$

6. $(x + 5)(3x^2 - x + 1)$

7. $(2x^4 - 5x^2)(6x + 4x^2)$

8. $(x + y)(2x - y)$

9. $(x + 2y)(x^2 + xy + y^2)$

10. $(x^3)(x^2 - 3)(3x + 1)$

11. The *vertex form* of a quadratic function is $f(x) = a(x - h)^2 + k$. Use your knowledge about multiplying binomials to complete the following.

$f(x) = a(x - h)\left(\square - \square\right) + k$ Write as a product of two binomials.

$= a\left(x^2 - \square x + \square^2\right) + k$ Multiply the binomials.

$= ax^2 - \square x + \square + k$ Distribute the constant *a*.

Compare this rewritten form to the standard form of a quadratic function, $f(x) = ax^2 + bx + c$. Discuss how b and c relate to the rewritten function. How can you rewrite a quadratic function in vertex form so that it is in standard form?

12. The set of polynomials is analogous to a set of numbers you have studied. To determine this set of numbers, consider the following questions about closure.

   a. Under which operations is the set of polynomials closed?

   b. Which set of the numbers discussed in the lesson on Rational Exponents is closed under the same set of operations?